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Calculator Free
Anti-Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] **CF**

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

(a) $\int \frac{4}{t^2}\,dt$

(b) $\int -\sin 2u\ du$

(c) $\int (4x-5)^3\ dx$

(d) $\int (e^{-5x} + 2\pi x - \sqrt{x})\ dx$

(e) $\int \frac{4t^6 - 6t^2}{8t^2}\ dt$

(f) $\int \left(x^2 - 2\right)^3\ dx$

(g) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2}\right) dx$

(h) $\int \left(e^{-2x} + 1\right)\left(e^{3x} - 2\right) dx$

Question Two: [3, 3, 3 = 9 marks] CF

Calculate the following integrals, showing all working.

(a) $\int_{-1}^{2}\left(x^2-1\right)dx$

(b) $-2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sin 3x\ dx$

(c) $\int_{-1}^{3}\left(-e^{4x}+2\right)dx$

Question Three: [3 marks] CF

The derivative of $f(x)$ is given by $f'(x) = 2e^{2x} + 3x^2$. Given that $f(1) = 4 + e^2$, find an expression for $f(x)$.

Question Four: [6 marks] CF

The gradient function of $f(x)$ is given by $f'(x) = ax^2 + b$. Determine the values of a and b if $f'(-2) = 28,\ f(0) = 1$ and $f(1) = 7$.

Question Five: [1, 2, 3 = 6 marks] CF

Given that $\int_{-1}^{2} f(x)\,dx = 4$ and $\int_{-1}^{7} f(x)\,dx = 10$, determine:

(a) $2\int_{-1}^{7} f(x)\,dx$

(b) $\int_{7}^{2} f(x)\,dx$

(c) $\int_{-1}^{2} \left(f(x) + x \right) dx$

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Question One: [2, 2, 2, 3, 3, 3, 3, 3 = 21 marks] **CF**

Anti-differentiate each of the following, showing all working. Leave all answers with positive indices.

(a) $\int \frac{4}{t^2}\,dt$

$\int 4t^{-2}dt$

$= \frac{4t^{-1}}{-1} + c$ ✓

$= \frac{-4}{t} + c$ ✓

(b) $\int -\sin 2u \; du$

$= \frac{\cos 2u}{2} + c$ ✓ ✓

(c) $\int (4x-5)^3 \; dx$

$= \frac{(4x-5)^4}{4\times 4} + c$ ✓

$= \frac{(4x-5)^4}{16} + c$ ✓

(d) $\int (e^{-5x} + 2\pi x - \sqrt{x}) \; dx$

$= \frac{e^{-5x}}{-5} + \frac{2\pi x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$

$= \frac{-1}{5e^{5x}} + \frac{2\pi x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$

✓ ✓ ✓

(e) $\int \frac{4t^6 - 6t^2}{8t^2} \, dt$

$= \int \frac{4t^6}{8t^2} - \frac{6t^2}{8t^2} \, dt$ ✓

$= \int \frac{t^4}{2} - \frac{3}{4} \, dt$

$= \frac{t^5}{10} - \frac{3t}{4} + c$

✓ ✓

(f) $\int \left(x^2 - 2\right)^3 \, dx$

$= \int \left(x^6 + 3(x^2)^2(-2) + 3(x^2)(-2)^2 + (-2)^3\right) dx$

$= \int \left(x^6 - 6x^4 + 12x^2 - 8\right) dx$ ✓

$= \frac{x^7}{7} - \frac{6x^5}{5} + \frac{12x^3}{3} - 8x + c$

✓ ✓

(g) $\int \left(\cos\left(\frac{x}{3}\right) + \frac{\sqrt[3]{6x}}{2} \right) dx$

✓ ✓

$= 3\sin\left(\frac{x}{3}\right) + \frac{3(6x)^{\frac{4}{3}}}{8} + c$

✓

(h) $\int \left(e^{-2x} + 1\right)\left(e^{3x} - 2\right) dx$

$= \int \left(e^x - 2e^{-2x} + e^{3x} - 2\right) dx$ ✓

$= e^x + \frac{1}{e^{2x}} + \frac{e^{3x}}{3} - 2x + c$

✓ ✓

Question Two: [3, 3, 3 = 9 marks] CF

Calculate the following integrals, showing all working.

(a) $\int_{-1}^{2}\left(x^2-1\right)dx$

$$=\left[\frac{x^3}{3}-x\right]_{-1}^{2} \checkmark$$

$$=\left(\frac{8}{3}-2\right)-\left(\frac{-1}{3}+1\right) \checkmark$$

$$=\frac{9}{3}-3$$

$$=3-3$$

$$=0 \checkmark$$

(b) $-2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sin 3x\ dx$

$$=-2\left[\frac{-\cos 3x}{3}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \checkmark$$

$$=-2\left[\frac{-\cos\pi}{3}-\frac{-\cos\frac{\pi}{2}}{3}\right] \checkmark$$

$$=-2\left(\frac{1}{3}+0\right)$$

$$=\frac{-2}{3} \checkmark$$

(c) $\int_{-1}^{3}\left(-e^{4x}+2\right)dx$

$$=\left[\frac{-e^{4x}}{4}+2x\right]_{-1}^{3} \checkmark$$

$$=\left(\frac{-e^{12}}{4}+6\right)-\left(\frac{-e^{-4}}{4}-2\right) \checkmark$$

$$=\frac{-e^{12}+e^{4}}{4}+8 \checkmark$$

Question Three: [3 marks] CF

The derivative of $f(x)$ is given by $f'(x) = 2e^{2x} + 3x^2$. Given that $f(1) = 4 + e^2$, find an expression for $f(x)$.

$f(x) = \int 2e^{2x} + 3x^2 \, dx$

$f(x) = e^{2x} + x^3 + c$ ✓

$4 + e^2 = e^2 + 1 + c$ ✓

$c = 3$

$f(x) = e^{2x} + x^3 + 3$ ✓

Question Four: [6 marks] CF

The gradient function of $f(x)$ is given by $f'(x) = ax^2 + b$. Determine the values of a and b if $f'(-2) = 28$, $f(0) = 1$ and $f(1) = 7$.

$28 = 4a + b$ ✓

$f(x) = \frac{ax^3}{3} + bx + c$ ✓

$1 = c$ ✓

$7 = \frac{a}{3} + b + 1$ ✓

$6 = \frac{a}{3} + b$

$28 = 4a + b$

$22 = \frac{11}{3}a$

$\frac{66}{11} = a$

$6 = a$ ✓

$28 = 24 + b$

$b = 4$ ✓

Question Five: **[1, 2, 3 = 6 marks]** **CF**

Given that $\int_{-1}^{2} f(x)\, dx = 4$ and $\int_{-1}^{7} f(x)\, dx = 10$, determine:

(a) $2\int_{-1}^{7} f(x)\, dx$

$= 2 \times 10$

$= 20$ ✓

(b) $\int_{7}^{2} f(x)\, dx$

$= \int_{-1}^{7} f(x)dx - \int_{-1}^{2} f(x)dx$ ✓

$= 10 - 4$

$= 6$

$\therefore -6$ ✓

(c) $\int_{-1}^{2} \left(f(x) + x \right) dx$

$= \int_{-1}^{2} f(x)dx + \int_{-1}^{2} x\, dx$ ✓

$= 4 + \left[\frac{x^2}{2} \right]_{-1}^{2}$

$= 4 + \left(\frac{4}{2} - \frac{1}{2} \right)$ ✓

$= 4 + \frac{3}{2}$

$= 5\frac{1}{2}$ ✓